Numeric Response Questions

Inverse Trigonometric Functions

Q.1 If $\sin^{-1}\ \sin\ 17 + \cos^{-1}\ \cos\ 27 + \tan^{-3}\ \tan\ 37$ is equal to $k - \lambda\pi$ then find $k + \lambda$.

Q.2 If the value of $\cot^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13}$ is $\sin^{-1}\left(\frac{a}{b}\right)$ then find $b - a$.

Q.3 Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.

Q.4 If $\sum_{i=1}^{20}\sin^{-1}\ x_i = 10\pi$ then find the value of $\sum_{i=1}^{20} x_i$,

Q.5 If $\sin^{-1}\ x + \sin^{-1}\ y + \sin^{-1}\ z = \frac{3\pi}{2}$, then find value of $\cos^{-1}\ x + \cos^{-1}\ y + \cos^{-1}\ z$

Q.6 If $(\tan^{-1}\ x)^2 + (\cot^{-1}\ x)^2 = \frac{5\pi^2}{8}$, then find value of x.

Q.7 If $\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \cos^{-1}\left(\frac{140}{k}\right)$ then find k.

Q.8 If $\tan^{-1}\ x + 2\cot^{-1}\ x = \frac{2\pi}{3}$, then find value of x.

Q.9 If $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$, then find value of x.

Q.10 If $\tan^{-1}\frac{x}{\pi} < \frac{\pi}{3}, x \in N$, then find the maximum value of x,

Q.11 If $2\sin^{-1}\ x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $|x| \leq$ a then find a.

Q.12 If the value of $\sin\ (\cos^{-1}\ x)$ is $\sqrt{1-x^n}$ then find value of n,

Q.13 If $\cos^{-1}\ p + \cos^{-1}\ q + \cos^{-1}\ r = \pi$, then find value of $p^2 + q^2 + r^2 + 2pqr$.

Q.14 If the value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is $\frac{k}{17}$ then find k.

Q.15 Find the value of $\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{xz+1}{x-z}$.

ANSWER KEY

1. 62.00	**2.** 2.00	**3.** 0.00	**4.** 20.00	**5.** 0.00	**6.** –1.00	**7.** 221.00
8. 1.73	**9.** 13.00	**10.** 5.00	**11.** 0.71	**12.** 2.00	**13.** 1.00	**14.** 6.00
15. 0.00						

Hints & Solutions

1. $\sin^{-1}\sin 17 = 5\pi - 17$

$\cos^{-1}\cos 27 = 27 - 8\pi$

$\tan^{-1}\tan 37 = \underline{37 - 12\pi}$

$= 47 - 15\pi$

2. $\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12}$

$= \tan^{-1}\left(\frac{48+15}{36-20}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

$= \sin^{-1}\frac{63}{65}$

3. $\cos^{-1}\cos 300^\circ + \sin^{-1}\sin 300^\circ$

$= \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} - \frac{\pi}{3} = 0$

4. $\frac{-\pi}{2} \le \sin^{-1}x_i \le \frac{\lambda}{2}$ $\quad \because \sin^{-1}x_i = \frac{\pi}{2}$

$x_i = 1$ $\quad \because \ 1 \le i \le 20$

so , $\sum_{i=1}^{20} x_i = 20$

5. $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \cos^{-1}y = \cos^{-1}z = 0$

6. We have $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow (\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8}$

$\Rightarrow \frac{\pi^2}{4} - 2.\frac{\pi}{2}\tan^{-1}x + 2(\tan^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$

$\Rightarrow \tan^{-1}x = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow \tan^{-1}x = -\frac{\pi}{4} \Rightarrow x = -1$

7. $\cos^{-1}\frac{15}{17} = \tan^{-1}\frac{8}{15}$

$2\tan^{-1}\left(\frac{1}{15}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

$\therefore \cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$

$= \tan^{-1}\left(\frac{8}{15}\right) + \tan^{-1}\left(\frac{5}{12}\right)$

$= \tan^{-1}\left(\frac{\frac{8}{15}+\frac{5}{12}}{1-\frac{40}{180}}\right) = \tan^{-1}\left(\frac{171}{140}\right)$

$= \cos^{-1}\left(\frac{140}{221}\right)$

8. $\frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}$

$\cot^{-1}x = \frac{\pi}{6}$

$x = \cot\frac{\pi}{6} = \sqrt{3}$

9. $\cos^{-1}\frac{\sqrt{x^2-25}}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

$\sqrt{x^2-25} = 12$

$x^2 = 169$

$x = 13$

10. $\tan^{-1}\left(\frac{x}{\pi}\right) < \frac{\pi}{3}$

$\tan\tan^{-1}\left(\frac{x}{\pi}\right) < \tan\frac{\pi}{3}$

$x < \sqrt{3}\,\pi$

$x = 5.5$ (approximate)

11. Let $x = \sin\theta \Rightarrow -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$

$-\pi \le 2\theta \le \pi$

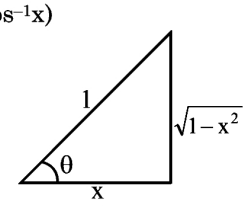
only when $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$-\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$

or $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

12. $\sin(\cos^{-1}x)$



$\sin(\cos^{-1}x) = \sqrt{1-x^2}$ for all x.

13. $\cos^{-1} + \cos^{-1}q + \cos^{-1}r = \pi$

Let $\cos^{-1}p = \cos^{-1}q = \cos^{-1}r = \frac{\pi}{3}$

$\Rightarrow p = q = r = \cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore\ p^2 + q^2 + r^2 + 2pqr$

$= \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1$

14. $\cot\left[\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right]$

$\cot\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$

$= \cot\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\right]$

$= \cot\left[\tan^{-1}\left(\frac{\frac{9+8}{12}}{\frac{12-6}{12}}\right)\right]$

$= \cot\left[\tan^{-1}\frac{17}{6}\right]$

$= \cot\left[\cot^{-1}\frac{6}{17}\right] = \frac{6}{17}$

15. $\tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \tan^{-1}\left(\frac{x-z}{1+xz}\right)$

$= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}x - \tan^{-1}z$

$= 0$